30 *Years*

Previous Solved Papers

GATE 2024

Mechanical Engineering

- Fully solved with explanations
- Analysis of previous papers
- Topicwise presentation
- Thoroughly revised & updated

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GATE - 2024

Mechanical Engineering

Topicwise Previous GATE Solved Papers (1994-2023)

Editions

1st Edition	:	2008
2nd Edition	:	2009
3rd Edition	:	2010
4th Edition	:	2011
5th Edition	:	2012
6th Edition	:	2013
7th Edition	:	2014
8th Edition	:	2015
9th Edition	:	2016
10th Edition	:	2017
11th Edition	:	2018
12th Edition	:	2019
13th Edition	:	2020
14th Edition	:	2021
15th Edition	:	2022
16th Edition	**:**	**2023**

Preface

B. Singh (Ex. IES)

Over the period of time the GATE examination has become more challenging due to increasing number of candidates. Though every candidate has ability to succeed but competitive environment, in-depth knowledge, quality guidance and good source of study is required to achieve high level goals.

The new edition of **GATE 2024 Solved Papers : Mechanical Engineering** has been fully revised, updated and edited. The whole book has been divided into topicwise sections.

At the beginning of each subject, analysis of previous papers are given to improve the understanding of subject.

I have true desire to serve student community by way of providing good source of study and quality guidance. I hope this book will be proved an important tool to succeed in GATE examination. Any suggestions from the readers for the improvement of this book are most welcome.

B. Singh (Ex. IES)
Chairman and Managing Director
MADE EASY Group

GATE-2024

Mechanical Engineering

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Engineering Mathematics

UNIT I

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Engineering Mathematics

Syllabus

***Linear Algebra* :** Matrix algebra, systems of linear equations, eigenvalues and eigenvectors.

***Calculus*:** Functions of single variable, limit, continuity and differentiability, mean value theorems, indeterminate forms; evaluation of definite and improper integrals; double and triple integrals; partial derivatives, total derivative, Taylor series (in one and two variables), maxima and minima, Fourier series; gradient, divergence and curl, vector identities, directional derivatives, line, surface and volume integrals, applications of Gauss, Stokes and Green's theorems.

***Differential equations*:** First order equations (linear and nonlinear); higher order linear differential equations with constant coefficients; Euler-Cauchy equation; initial and boundary value problems; Laplace transforms; solutions of heat, wave and Laplace's equations.

***Complex variables*:** Analytic functions; Cauchy-Riemann equations; Cauchy's integral theorem and integral formula; Taylor and Laurent series.

***Probability and Statistics*:** Definitions of probability, sampling theorems, conditional probability; mean, median, mode and standard deviation; random variables, binomial, Poisson and normal distributions.

***Numerical Methods*:** Numerical solutions of linear and non-linear algebraic equations; integration by trapezoidal and Simpson's rules; single and multi-step methods for differential equations.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	3	6	**15**
2004	3	5	**13**
2005	5	10	**25**
2006	4	8	**20**
2007	4	8	**20**
2008	6	9	**24**
2009	4	8	**20**
2010	5	3	**11**
2011	5	4	**13**
2012	5	5	**15**
2013	5	5	**15**
2014 Set-1	5	4	**13**
2014 Set-2	5	4	**13**
2014 Set-3	5	4	**13**
2014 Set-4	5	4	**13**
2015 Set-1	4	3	**10**
2015 Set-2	4	4	**12**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2015 Set-3	6	5	**16**
2016 Set-1	5	4	**13**
2016 Set-2	5	4	**13**
2016 Set-3	5	4	**13**
2017 Set-1	5	4	**13**
2017 Set-2	4	4	**12**
2018 Set-1	5	4	**13**
2018 Set-2	5	4	**13**
2019 Set-1	5	4	**13**
2019 Set-2	5	4	**13**
2020 Set-1	6	4	**14**
2020 Set-2	5	4	**13**
2021 Set-1	5	3	**11**
2021 Set-2	5	4	**13**
2022 Set-1	5	4	**13**
2022 Set-2	5	4	**13**
2023	5	4	**13**

1 Linear Algebra

1.1 For the matrix $\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ the eigen values are

(a) 3 and –3 (b) –3 and –5
(c) 3 and 5 (d) 5 and 0 **[2003 : 1 M]**

1.2 Consider the system of simultaneous equations

$$x + 2y + z = 6$$
$$2x + y + 2z = 6$$
$$x + y + z = 5$$

This system has

(a) unique solution (b) infinite number of solutions
(c) no solution (d) exactly two solutions

[2003 : 2 M]

1.3 The sum of the eigen values of the matrix given below is

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

(a) 5 (b) 7
(c) 9 (d) 18 **[2004 : 1 M]**

1.4 For which value of x will the matrix given below become singular?

$$\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

(a) 4 (b) 6
(c) 8 (d) 12 **[2004 : 2 M]**

1.5 A is a 3 × 4 real matrix and $Ax = B$ is an inconsistent system of equations. The highest possible rank of A is

(a) 1 (b) 2
(c) 3 (d) 4 **[2005 : 1 M]**

1.6 Which one of the following is an eigenvector of the matrix

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}$ **[2005 : 2 M]**

1.7 With a 1 unit change in b, what is the change in x in the solution of the system of equations $x + y = 2$, $1.01\,x + 0.99\,y = b$?

(a) zero (b) 2 units
(c) 50 units (d) 100 units

[2005 : 2 M]

1.8 Let x denotes a real number. Find out the INCORRECT statement.

(a) $S = \{x : x > 3\}$ represents the set of all real numbers greater than 3
(b) $S = \{x : x^2 < 0\}$ represents the empty set.
(c) $S = \{x : x \in A$ and $x \in B\}$ represents the union of set A and set B.
(d) $S = \{x : a < x < b\}$ represents the set of all real numbers between a and b, where a and b are real numbers.

[2006 : 1 M]

1.9 Eigenvalues of a matrix $S = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are 5 and 1. What are the eigenvalues of the matrix $S^2 = SS$?

(a) 1 and 25 (b) 6 and 4
(c) 5 and 1 (d) 2 and 10

[2006 : 2 M]

1.10 Match the items in columns I and II.

Column I

P. Singular matrix
Q. Non-square matrix
R. Real symmetric
S. Orthogonal matrix

Column II

1. Determinant is not defined
2. Determinant is always one
3. Determinant is zero
4. Eigenvalues are always real
5. Eigenvalues are not defined

(a) P–3, Q–1, R–4, S–2
(b) P–2, Q–3, R–4, S–1
(c) P–3, Q–2, R–5, S–4
(d) P–3, Q–4, R–2, S–1 [2006 : 2 M]

1.11 Multiplication of matrices E and F is G. Matrices E and G are

$$E = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What is the matrix F?

(a) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} \cos\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} \sin\theta & -\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ [2006 : 2 M]

1.12 If a square matrix A is real and symmetric, then the eigenvalues
(a) are always real
(b) are always real and positive
(c) are always real and non-negative
(d) occur in complex conjugate pairs
[2007 : 1 M]

1.13 The number of linearly independent eigenvectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is
(a) 0 (b) 1
(c) 2 (d) infinite [2007 : 2 M]

1.14 The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ has one eigenvalue equal to 3. The sum of the other two eigenvalues is
(a) p (b) $p - 1$
(c) $p - 2$ (d) $p - 3$ [2008 : 1 M]

1.15 For what value of a, if any, will the following system of equations in x, y and z have a solution?

$$2x + 3y = 4$$
$$x + y + z = 4$$
$$x + 2y - z = a$$

(a) Any real number (b) 0
(c) 1 (d) There is no such value
[2008 : 2 M]

1.16 The eigenvectors of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ are written in the form $\begin{bmatrix} 1 \\ a \end{bmatrix}$ and $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is $a + b$?
(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) 2 [2008 : 2 M]

1.17 For a matrix $[M] = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$, the transpose of the matrix is equal to the inverse of the matrix, $[M]^T = [M]^{-1}$. The value of x is given by
(a) $-\frac{4}{5}$ (b) $-\frac{3}{5}$
(c) $\frac{3}{5}$ (d) $\frac{4}{5}$ [2009 : 1 M]

1.18 One of the eigen vectors of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ is
(a) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$ (b) $\begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$
(c) $\begin{Bmatrix} 4 \\ 1 \end{Bmatrix}$ (d) $\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$ [2010 : 2 M]

1.19 Eigenvalues of a real symmetric matrix are always
(a) positive (b) negative
(c) real (d) complex [2011 : 1 M]

1.20 Consider the following system of equations

$$2x_1 + x_2 + x_3 = 0$$
$$x_2 - x_3 = 0$$
$$x_1 + x_2 = 0$$

This system has
(a) a unique solution
(b) no solution
(c) infinite number of solutions
(d) five solutions [2011 : 2 M]

1.21 For the matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, ONE of the normalized eigen vectors is given as

(a) $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

(c) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$ **[2012 : 2 M]**

1.22 $x + 2y + z = 4$
$2x + y + 2z = 5$
$x - y + z = 1$
The system of algebraic given below has
(a) A unique solution of $x = 1$, $y = 1$ and $z = 1$
(b) only the two solutions of ($x = 1$, $y = 1$, $z = 1$) and ($x = 2$, $y = 1$, $z = 0$)
(c) infinite number of solutions
(d) no feasible solution **[2012 : 2 M]**

1.23 Choose the CORRECT set of functions, which are linearly dependent.
(a) $\sin x$, $\sin^2 x$ and $\cos^2 x$
(b) $\cos x$, $\sin x$ and $\tan x$
(c) $\cos 2x$, $\sin^2 x$ and $\cos^2 x$
(d) $\cos 2x$, $\sin x$ and $\cos x$ **[2013 : 1 M]**

1.24 The eigen values of a symmetric matrix are all
(a) complex with non-zero positive imaginary part
(b) complex with non-zero negative imaginary part
(c) real
(d) pure imaginary **[2013 : 1 M]**

1.25 Given that the determinant of the matrix $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$ is – 12, the determinant of the matrix $\begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix}$ is

(a) – 96 (b) – 24
(c) 24 (d) 96 **[2014 : 1 M, Set-1]**

1.26 The matrix form of the linear system $\frac{dx}{dt} = 3x - 5y$ and $\frac{dy}{dt} = 4x + 8y$ is

(a) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & -5 \\ 4 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(b) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & 8 \\ 4 & -5 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(c) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 4 & -5 \\ 3 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(d) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 4 & 8 \\ 3 & -5 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$ **[2014 : 1 M, Set-1]**

1.27 One of the eigenvectors of matrix $\begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ is

(a) $\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$ (b) $\begin{Bmatrix} -2 \\ 9 \end{Bmatrix}$

(c) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$ (d) $\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

[2014 : 1 M, Set-2]

1.28 Consider a 3 × 3 real symmetric matrix S such that two of its eigenvalues are $a \neq 0$, $b \neq 0$ with respective eigenvectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$. If $a \neq b$ then $x_1y_1 + x_2y_2 + x_3y_3$ equals
(a) a (b) b
(c) ab (d) 0 **[2014 : 1 M, Set-3]**

1.29 Which one of the following equations is a correct identity for arbitrary 3 × 3 real matrices P, Q and R?
(a) $P(Q + R) = PQ + RP$
(b) $(P - Q)^2 = P^2 - 2PQ + Q^2$
(c) $\det(P + Q) = \det P + \det Q$
(d) $(P + Q)^2 = P^2 + PQ + QP + Q^2$
[2014 : 1 M, Set-4]

1.30 If any two columns of a determinant $P = \begin{vmatrix} 4 & 7 & 8 \\ 3 & 1 & 5 \\ 9 & 6 & 2 \end{vmatrix}$ are interchanged, which one of the following statements regarding the value of the determinant is CORRECT?
(a) Absolute value remains unchanged but sign will change
(b) Both absolute value and sign will change
(c) Absolute value will change but sign will not change
(d) Both absolute value and sign will remain unchanged **[2015 : 1 M, Set-1]**

1.31 At least one eigenvalue of a singular matrix is
(a) positive (b) zero
(c) negative (d) imaginary

[2015 : 1 M, Set-3]

1.32 The lowest eigenvalue of the 2 × 2 matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ is ______.

[2015 : 1 M, Set-3]

1.33 For given matrix $P = \begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$ where $i = \sqrt{-1}$, the inverse of matrix P is

(a) $\frac{1}{24}\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}$ (b) $\frac{1}{25}\begin{bmatrix} i & 4-i \\ 4+3i & -i \end{bmatrix}$

(c) $\frac{1}{24}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$ (d) $\frac{1}{25}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$

[2015 : 2 M, Set-3]

1.34 The solution to the system of equations is

$$\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -30 \end{bmatrix}$$

(a) 6, 2 (b) –6, 2
(c) –6, –2 (d) 6, – 2

[2016 : 1 M, Set-1]

1.35 The condition for which the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$ are positive, is

(a) $k > \frac{1}{2}$ (b) $k > -2$

(c) $k > 0$ (d) $k < -\frac{1}{2}$

[2016 : 1 M, Set-2]

1.36 A real square matrix A is called skew-symmetric if
(a) $A^T = A$ (b) $A^T = A^{-1}$
(c) $A^T = -A$ (d) $A^T = A + A^{-1}$

[2016 : 1 M, Set-3]

1.37 The number of linearly independent eigenvectors of matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is________

[2016 : 2 M, Set-3]

1.38 The product of eigenvalues of the matrix P is

$$P = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

(a) – 6 (b) 2
(c) 6 (d) – 2

[2017 : 1 M, Set-1]

1.39 Consider the matrix $P = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$

Which one of the following statements about P is INCORRECT?
(a) Determinant of P is equal to 1.
(b) P is orthogonal.
(c) Inverse of P is equal to its transpose.
(d) All eigenvalues of P are real numbers.

[2017 : 2 M, Set-1]

1.40 The determinant of a 2 × 2 matrix is 50. If one eigenvalue of the matrix is 10, the other eigen value is ___________.

[2017 : 1 M, Set-2]

1.41 Consider the matrix $A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix}$ whose eigenvectors corresponding to eigenvalues λ_1 and λ_2 are $x_1 = \begin{bmatrix} 70 \\ \lambda_1 - 50 \end{bmatrix}$ and $x_2 = \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix}$, respectively. The value of $x_1^T x_2$ is __________.

[2017 : 2 M, Set-2]

1.42 The rank of the matrix $\begin{bmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{bmatrix}$ is
(a) 1 (b) 2
(c) 3 (d) 4

[2018 : 1 M, Set-1]

1.43 If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ then $\det(A^{-1})$ is _______ (correct to two decimal places).

[2018 : 1 M, Set-2]

1.44 Consider the matrix

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The number of distinct eigen values of P is
(a) 2 (b) 0
(c) 1 (d) 3 **[2019 : 1 M, Set-1]**

1.45 The set of equations
$x + y + z = 1$
$ax - ay + 3z = 5$
$5x - 3y + az = 6$
has infinite solution if a =
(a) – 4 (b) – 3
(c) 3 (d) 4 **[2019 : 2 M, Set-1]**

1.46 In matrix equation $[A]\{X\} = \{R\}$

$$[A] = \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}, \{X\} = \begin{Bmatrix} 2 \\ 1 \\ 4 \end{Bmatrix} \text{ and } \{R\} = \begin{Bmatrix} 32 \\ 16 \\ 64 \end{Bmatrix}$$

One of the eigen values of Matrix $[A]$ is
(a) 4 (b) 15
(c) 8 (d) 16
[2019 : 1 M, Set-2]

1.47 Multiplication of real valued square matrices of same dimension is
(a) not always possible to compute
(b) associative
(c) always positive definite
(d) commutative **[2020 : 1 M, Set-1]**

1.48 A matrix P is decomposed into its symmetric part S and skew symmetric part V. If

$$S = \begin{pmatrix} -4 & 4 & 2 \\ 4 & 3 & 7/2 \\ 2 & 7/2 & 2 \end{pmatrix}, \quad V = \begin{pmatrix} 0 & -2 & 3 \\ 2 & 0 & 7/2 \\ -3 & -7/2 & 0 \end{pmatrix}$$

then matrix P is

(a) $\begin{pmatrix} -2 & 9/2 & -1 \\ -1 & 81/4 & 11 \\ -2 & 45/2 & 73/4 \end{pmatrix}$ (b) $\begin{pmatrix} -4 & 6 & -1 \\ 2 & 3 & 0 \\ 5 & 7 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 4 & -6 & 1 \\ -2 & -3 & 0 \\ -5 & -7 & -2 \end{pmatrix}$

[2020 : 1 M, Set-2]

1.49 Let $\boldsymbol{I}$ be a 100 dimensional identity matrix and E be the set of its distinct (no value appears more than once in $\boldsymbol{E}$) real eigenvalues. The number of elements in $\boldsymbol{E}$ is ______.
[2020 : 1 M, Set-2]

1.50 Consider an $n \times n$ matrix A and a non-zero $n \times 1$ vector p. Their product $Ap = \alpha^2 p$, where $\alpha \in \Re$ and $\alpha \notin \{-1, 0, 1\}$. Based on the given information, the eigen value of A^2 is:
(a) $\sqrt{\alpha}$ (b) α^2
(c) α (d) α^4
[2021 : 1 M, Set-2]

1.51 Let the superscript T represent the transpose operation. Consider the function $f(x) = \frac{1}{2}x^T Qx - r^T x$, where x and r are $n \times 1$ vectors and Q is a symmetric n × n matrix. The stationary point of $f(x)$ is
(a) $\frac{r}{r^T r}$ (b) $Q^T r$
(c) $Q^{-1} r$ (d) r **[2021 : 2 M, Set-2]**

1.52 Consider a vector p in 2-dimensional space. Let its direction (counter-clockwise angle with the positive x-axis) be θ. Let p be an eigen vector of a 2 × 2 matrix A with corresponding eigen value λ, $\lambda > 0$. If we denote the magnitude of a vector v by $\|v\|$, identify the VALID statement regarding p', where $p' = Ap$.
(a) Direction of $p' = \theta, \|p'\| = \lambda\|p\|$
(b) Direction of $p' = \lambda\theta, \|p'\| = \|p\|$
(c) Direction of $p' = \theta, \|p'\| = \|p\|/\lambda$
(d) Direction of $p' = \lambda\theta, \|p'\| = \lambda\|p\|$
[2021 : 2 M, Set-1]

1.53 If $A = \begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix}$ is a symmetric matrix, the value of k is _______.
(a) 8 (b) 5
(c) –0.4 (d) $\frac{1+\sqrt{1561}}{12}$
[2022 : 1 M, Set-1]

1.54 The system of linear equations in real (x, y) given by

$$(x \;\; y)\begin{bmatrix} 2 & 5-2\alpha \\ \alpha & 1 \end{bmatrix} = (0 \;\; 0)$$

involves a real parameter α and has infinitely many non-trivial solutions for special value(s) of

α. Which one or more among the following options is/are non-trivial solution(s) of (x, y) for such special value(s) of α?

(a) $x = 2,\ y = -2$ (b) $x = -1,\ y = 4$
(c) $x = 1,\ y = 1$ (d) $x = 4,\ y = -2$

[2022 : 2 M, Set-1]

1.55 A is a 3×5 real matrix of rank 2. For the set of homogeneous equations $Ax = 0$, where 0 is a zero vector and x is a vector of unknown variables, which of the following is/are true?

(a) The given set of equations will have a unique solution.
(b) The given set of equations will be satisfied by a zero vector of appropriate size.
(c) The given set of equations will have infinitely many solutions.
(d) The given set of equations will have many but a finite number of solutions.

[2022 : 2 M, Set-2]

1.56 If the sum and product of eigen values of a 2×2 matrix $\begin{bmatrix} 3 & p \\ p & q \end{bmatrix}$ are 4 and –1 respectively, then $|p|$ is ______ (in integer).

[2022 : 2 M, Set-2]

1.57 A linear transformation maps a point (x, y) in the plane to the point $(\hat{x}, \hat{y})$ according to the rule

$\hat{x} = 3y,\ \hat{y} = 2x$

Then, the disc $x^2 + y^2 \le 1$ gets transformed to a region with an area equal to ______. (Rounded off to two decimals)

Use $\pi = 3.14$.

[2023 : 1 M]

Answers Linear Algebra

1.1	(c)	1.2	(c)	1.3	(b)	1.4	(a)	1.5	(b)	1.6	(a)	1.7	(c)	1.8	(c)
1.9	(a)	1.10	(a)	1.11	(c)	1.12	(a)	1.13	(c)	1.14	(c)	1.15	(b)	1.16	(b)
1.17	(a)	1.18	(a)	1.19	(c)	1.20	(c)	1.21	(b)	1.22	(c)	1.23	(c)	1.24	(c)
1.25	(a)	1.26	(a)	1.27	(d)	1.28	(d)	1.29	(d)	1.30	(a)	1.31	(b)	1.32	(2)
1.33	(a)	1.34	(d)	1.35	(a)	1.36	(c)	1.37	(2)	1.38	(b)	1.39	(d)	1.40	(5)
1.41	(0)	1.42	(b)	1.43	$(\frac{1}{4})$	1.44	(c)	1.45	(d)	1.46	(d)	1.47	(b)	1.48	(c)
1.49	(1)	1.50	(d)	1.51	(c)	1.52	(a)	1.53	(a)	1.54	(a, b)	1.55	(b, c)	1.56	(2)
1.57	(18.84)														

Explanations Linear Algebra

1.1 (c)

$$A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

Now $|A - \lambda I| = 0$

where $\lambda =$ eigen value

$$\therefore \quad \begin{vmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)^2 - 1 = 0$$

or, $(4-\lambda)^2 - (1)^2 = 0$

or, $(4 - \lambda + 1)(4 - \lambda - 1) = 0$

or, $(5-\lambda)(3-\lambda) = 0$

$\therefore \quad \lambda =$ **3, 5**

1.2 (c)

Given equation are

$$x + 2y + z = 6$$
$$2x + y + 2z = 6$$
$$x + y + z = 5$$

Given system can be written as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 5 \end{bmatrix}$$

Applying row operation

$R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ -1 \end{bmatrix}$$

and applying $R_3 \to 3R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ 3 \end{bmatrix}$$

Since the rank of coefficient matrix is 2 and rank of argument matrix is 3, which are not equal. Hence system has **no solution**.

1.3 (b)

Sum of eigen values of given matrix
= sum of diagonal element of given matrix
= 1 + 5 + 1 **= 7**

1.4 (a)

For singularity of matrix $= \begin{vmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{vmatrix} = 0$

$\Rightarrow 8\,(0 - 12) - x\,(0 - 2 \times 12) = 0$

$\therefore \quad x = \mathbf{4}$

1.5 (b)

$C = [A : B]_{3 \times 5}$

$\therefore \quad \rho[C_{3\times 5}] \le \min\{3, 5\}$

& $\because$ The system is inconsistent

$\rho(A) < \rho(C)$

$\therefore \quad \rho(A) < 3$

Hence maximum possible rank of

$A = \mathbf{2}$

1.6 (a)

Eigen vector is an independent vector. Here in option *a* last two terms are zero. Hence only this option is correct.

1.7 (c)

Given $\quad x + y = 2$ ...(i)

$1.01\,x + 0.99\,y = b$ ...(ii)

Multiply 0.99 is equation (i), and substract from equation (ii); we get

$(1.01 - 0.99)\,x = b - 2 \times 0.99$

$0.02x = b - 1.98$

$\therefore \quad 0.02\,\Delta x = \Delta b$

$\therefore \quad \Delta x = \dfrac{1}{0.02}$ = **50 unit**

1.9 (a)

If $\lambda_1, \lambda_2, \lambda_3 \ldots \lambda_4$ are the eigen values of *A*. Then the eigen values of

A^m are $\lambda_1^m, \lambda_2^m, \lambda_3^m \ldots$

$\Rightarrow \quad S^2$ are $S_1^1, S_2^2, S_3^2 \ldots$

S matrix has eigen values 1 and *S*.

$\Rightarrow \quad S^2$ matrix has eigen values 1^2 and S^2

i.e. **1 and 25**

1.11 (c)

$$E = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $\quad G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The according to problem

$E \times F = G$

or $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence we see that product of $(E \times F)$ is unit matrix so *F* will be the inverse of *E*.

$$F = E^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1.12 (a)

The eigen values of any real and symmetric matrix **are always real.**

1.13 (c)

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$|A - \lambda I| = 0$

$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix} = 0$

$\Rightarrow \quad (2-\lambda)^2 = 0$

$\Rightarrow \quad \lambda = \mathbf{2}$

No. of linearly independent eigen vectors
= No. of distinct eigen values

1.14 (c)

Sum of the eigen values of matrix is
= sum of diagonal values present in the matrix
$\therefore\ 1 + 0 + p = 3 + \lambda_2 + \lambda_3$

$\Rightarrow p + 1 = 3 + \lambda_2 + \lambda_3$

$\Rightarrow \lambda_2 + \lambda_3 = p + 1 - 3 = \mathbf{p - 2}$

1.15 (b)

Augmented matrix is

$$\left[\begin{array}{ccc|c} 2 & 3 & 0 & 4 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & -1 & a \end{array}\right] \begin{array}{l} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 - R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 0 & 1 & -2 & -4 \\ 1 & 1 & 1 & 4 \\ 0 & 1 & -2 & a-4 \end{array}\right] R_1 \to R_1 - R_3$$

$$\sim \left[\begin{array}{ccc|c} 0 & 0 & 0 & -a \\ 1 & 1 & 1 & 4 \\ 0 & 1 & -2 & a-4 \end{array}\right]$$

will have solution if $\boldsymbol{a = 0}$

as Rank A = Rank (aug. A)

1.16 (b)

$$\begin{vmatrix} (1-\lambda) & 2 \\ 0 & (2-\lambda) \end{vmatrix} = 0$$

$$\Rightarrow \quad (1-\lambda)(2-\lambda) = 0$$

$$\therefore \quad \lambda = 1, 2$$

Putting the value of $\lambda = 1$

$$\Rightarrow \quad \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ a \end{bmatrix} = 0$$

$$\Rightarrow \quad a = 0$$

putting the value of $\lambda = 2$

$$\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ b \end{bmatrix} = 0$$

$$\Rightarrow \quad b = \frac{1}{2}$$

$$\therefore \quad a + b = \frac{1}{2}$$

1.17 (a)

If $\quad A^T = A^{-1}$

then A is orthogonal matrix.

Therefore $A.A^{-1} = A^{-1} A = I$

and $A^T A = A A^T = I$

Since M is orthogonal matrix

$$M^T M = I$$

$$\begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \left(\frac{3}{5}\right)^2 + x^2 & \left(\frac{3}{5}\cdot\frac{4}{5}\right) + \frac{3}{5}x \\ \left(\frac{4}{5}\cdot\frac{3}{5}\right) + \frac{3}{5}\cdot x & \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ Compare both sides a_{12}

$$a_{12} = \left(\frac{3}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{3}{5}\right)x = 0$$

$$\Rightarrow \quad \frac{3}{5}x = -\frac{3}{5}\cdot\frac{4}{5}$$

$$\Rightarrow \quad x = -\frac{4}{5}$$

1.18 (a)

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

Characteristic equation of A is

$$\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(3-\lambda) - 2 = 0$$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$\lambda = 1, 4$$

The eigen value problem is $[A - \lambda I]\hat{x} = 0$

$$\begin{bmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Putting $\lambda = 1$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 = 0 \quad \text{... (i)}$$

$$x_1 + 2x_2 = 0 \quad \text{... (ii)}$$

Solution is $x_2 = k$, $x_1 = -2k$

$$\hat{X}_1 = \begin{bmatrix} -2k \\ k \end{bmatrix}$$

i.e. $\quad x_1 : x_2 = -2 : 1$

Since, choice (A) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is in same ratio of x_1 to x_2.

$\therefore$ Choice (A) is an eigen vector.

1.19 (c)

Eigen values of symmetric matrix are always real.

1.20 (c)

The Augmented matrix

$$[A \mid B] = \left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right]$$

Performing gauss elimination on $[A \mid B]$ we get

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right] \xrightarrow{R_3 - \frac{1}{2}R_1} \left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right]$$

$$\xrightarrow{R_3-\frac{1}{2}R_2}\left[\begin{array}{ccc|c}2&1&1&0\\0&1&-1&0\\0&0&0&0\end{array}\right]$$

Rank (A) = Rank $(A \mid B)$ = **2 < 3**

So infinite number of solutions are obtained.

1.21 (b)

$$A = \begin{bmatrix}5&3\\1&3\end{bmatrix}$$

Characteristic equation is

$$\begin{vmatrix}5-\lambda&3\\1&3-\lambda\end{vmatrix} = 0$$

$$(5-\lambda)(3-\lambda)-3 = 0$$

$$\lambda^2-8\lambda+12 = 0$$

$$\lambda = 2, 6$$

Now, to find eigen vectors:

$$[A-\lambda I]\hat{x} = 0$$

Which is

$$\begin{bmatrix}5-\lambda&3\\1&3-\lambda\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

Put $\lambda = 2$ in above equation and we get

$$\begin{bmatrix}3&3\\1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

Which gives us the equation,

$$3x_1 + 3x_2 = 0$$

and $\qquad x_1 + x_2 = 0$

Which is only one equation,

$$x_1 + x_2 = 0$$

Whose solution is

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}-k\\k\end{bmatrix}$$

So one eigen vector is $\hat{x}_1 = \begin{bmatrix}-k\\k\end{bmatrix}$

Which after normalization is $= \dfrac{\hat{x}_1}{|\hat{x}_1|}$

$$= \frac{1}{\sqrt{(-k)^2+(k^2)}}\begin{bmatrix}-k\\k\end{bmatrix} = \begin{bmatrix}-\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix}$$

The other eigen vector is obtained by putting the other eigen value

$\lambda = 6$ in eigen value problem

$$\begin{bmatrix}5-\lambda&3\\1&3-\lambda\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

Which gives,

$$\begin{bmatrix}-1&3\\1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

Which gives the equation

$$-x_1 + 3x_2 = 0$$

and $\qquad x_1 - 3x_2 = 0$

Which is only one equation

$$-x_1 + 3x_2 = 0$$

Whose solution is

$$\hat{x}_2 = \begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}3k\\k\end{bmatrix}$$

Which after normalization is

$$\frac{\hat{x}_2}{|\hat{x}_2|} = \frac{1}{\sqrt{((3k)^2+k^2)}}\begin{bmatrix}3k\\k\end{bmatrix} = \begin{bmatrix}3/2\\1/2\end{bmatrix}$$

Choice (b) $\begin{bmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{bmatrix}$ is the only correct choice, since it is a constant multiple of one the normalized vectors which is $\hat{x}_1$.

1.22 (c)

The given system is

$$x + 2y + z = 4$$

$$2x + y + 2z = 5$$

$$x - y + z = 1$$

Use Gauss elimination method as follows:

Augmented matrix is

$$[A/B] = \left[\begin{array}{ccc|c}1&2&1&4\\2&1&2&5\\1&-1&1&1\end{array}\right]\xrightarrow[R_3-R_1]{R_2-2R_1}$$

$$\left[\begin{array}{ccc|c}1&2&1&4\\0&-3&0&-3\\0&-3&0&-3\end{array}\right]\xrightarrow{R_3-R_2}\left[\begin{array}{ccc|c}1&2&1&4\\0&-3&0&-3\\0&0&0&0\end{array}\right]$$

Rank (A) = 2

Rank (AB) = 2

So rank (A) = Rank (AB) = 2

System is consistent

Now system rank $r = 2$

Number of variables n = 3

$$r < n$$

So we have infinite number of solutions.

1.23 (c)

We know $\cos 2x = \cos^2 x - \sin^2 x$

$\therefore$ option (c) is correct.

1.24 (c)

(i) The Eigen values of symmetric matrix $[A^T = A]$ are purely real.

(ii) The Eigen value of skew-symmetric matrix $[A^T = -A]$ are either purely imaginary or zeros.

1.25 (a)

Let $D = -12$ for the given matrix

$$|A| = \begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix} = (2)^3 \begin{vmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{vmatrix}$$

(Taking 2 common from each row)

$\therefore \text{Det}(A) = (2)^3 \times D$

$= 8 \times -12 = \mathbf{-96}$

1.26 (a)

$$\frac{dx}{dt} = 3x - 5y$$

$$\frac{dy}{dt} = 4x + 8y$$

$$\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & -5 \\ 4 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3x & -5y \\ 4x & +8y \end{bmatrix}$$

1.27 (d)

The characteristic equation $|A - \lambda I| = 0$

i.e. $\begin{vmatrix} -5-\lambda & 2 \\ -9 & 6-\lambda \end{vmatrix} = 0$

or $(\lambda - 6)(\lambda + 5) + 18 = 0$

or $\lambda^2 - 6\lambda + 5\lambda - 30 + 18 = 0$

or $\lambda^2 - \lambda - 12 = 0$

or $\lambda = \dfrac{1 \pm \sqrt{1+48}}{2} = \dfrac{1 \pm 7}{2} = 4, -3$

Corresponding to $\lambda = 4$, we have

$$[A - \lambda I]x = \begin{bmatrix} -5-\lambda & 2 \\ -9 & 6-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

or, $\begin{bmatrix} -9 & 2 \\ -9 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

which gives only one independent equation, $-9x + 2y = 0$

$\therefore \dfrac{x}{2} = \dfrac{y}{9}$ gives eigen vector (2, 9)

Corresponding to $\lambda = -3$,

$$= \begin{bmatrix} -2 & 2 \\ -9 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

which gives $-x + y = 0$ (only one independent equation)

$\therefore \quad \dfrac{x}{1} = \dfrac{y}{1}$ which gives (1,1)

So, the eigen vectors are $\begin{Bmatrix} 2 \\ 9 \end{Bmatrix}$ and $\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$.

1.28 (d)

3 × 3 real symmetric matrix such that two of its eigen value are $a \neq 0$ $b \neq 0$ with respective eigen vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ if $a \neq b$ then

$x_1y_1 + x_2y_2 + x_3y_3 = \mathbf{0}$ because they are orthogonal.

$\therefore \quad x^Ty = 0 \quad$ (since $a \neq b$)

$$[x_1 \; x_2 \; x_3]\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = 0$$

$$x_1y_1 + x_2y_2 + x_3y_3 = \mathbf{0}$$

1.29 (d)

$$(P+Q)^2 = P^2 + PQ + QP + Q^2$$
$$= P.P + P.Q + Q.P + Q.Q$$
$$= \mathbf{P^2 + PQ + QP + Q^2}$$

1.30 (a)

Property of determinant : If any two row or column are interchanged, then magnitude of determinant remains same but sign changes.

1.31 (b)

For singular matrix

$|A| = 0$

According to properties of eigen value

Product of eigen values $= |A| = 0$

$\Rightarrow$ Atleast one of the eigen value is **zero**.

1.32 Sol.

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

$\therefore \quad |A - \lambda I| = 0$

$$\begin{bmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix} = (4-\lambda)(3-\lambda) - 2 = 0$$

$(\lambda - 4)(\lambda - 3) - 2 = 0$

$\lambda^2 - 7\lambda + 10 = 0$

$\Rightarrow \quad \lambda = 5, 2$

Minimum value = **2**

1.33 (a)

$$P = \begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$$

$$P^{-1} = \frac{\begin{bmatrix} 4-3i & -(-i) \\ -i & 4+3i \end{bmatrix}}{|A|} = \frac{1}{24}\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}$$

1.34 (d)

$$\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 & 2 \\ -4 & 3 & -30 \end{bmatrix}$$

$R_2 + 2R_1$

$$\begin{bmatrix} 2 & 5 & 2 \\ 0 & 13 & -26 \end{bmatrix}$$

$13y = -26$

or $y = -2$

$2x + 5y = 2$

$2x + 5(-2) = 2$

$2x = 2 + 10 = 12$

or $x = \mathbf{6}$

1.35 (a)

All Eigen values of $A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$ are positive

$2 > 0$

$\therefore$ 2×2 leading minor must be greater than zero

$$\begin{vmatrix} 2 & 1 \\ 1 & k \end{vmatrix} > 0$$

$2k - 1 > 0$

$2k > 1$

$k > \frac{1}{2}$

1.36 (c)

A is skew-symmetric

$\therefore \quad A^T = -A$

1.37 Sol.

Consider $A - \lambda I = \begin{bmatrix} 2-\lambda & 1 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

Ch. equation is $|A - \lambda I| = 0$

$(2-\lambda)(2-\lambda)(3-\lambda) = 0$

$\lambda = 2, 2, 3$

$\lambda = 3$ there is one L.I. Eigen vector

$\lambda = 2$ Consider $(A - 2I)\,x = 0$

rank = 2 The equation are $x_2 = 0$

No. of variables = 3 $x_3 = 0$

Let $x_1 = k$ be independent.

$\therefore$ Eigen vector is $\begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix} = k\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Only one independent Eigen vector in the case of $\lambda = 2$

Hence finally no. of L.I. Eigen vectors = 2

1.38 (b)

Product of Eigen value = determinant value

$= 2(3 - 6) + 1\,(8 - 0)$

$= 2(-3) + 8 = -6 + 8 = \mathbf{2}$

1.39 (d)

Explanations: The Eigen values of an orthogonal matrix are of magnitude 1 and are real (a) complex conjugate pairs.

1.40 Sol.

The product of eigen value is always equal to the determinant value of the matrix.

$\lambda_1 = 10 \quad \lambda_2 = \text{unknown} \quad |A| = 50$

$\lambda_1 \cdot \lambda_2 = 50$

$10\,(\lambda_2) = 50$

$\therefore \quad \lambda_2 = \mathbf{5}$

1.41 Sol.

$$A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix}$$

Eigen values of A are λ_1, λ_2

$\lambda_1 + \lambda_2 = 130$

$\lambda_1\lambda_2 = -900$

Given that $X_1 = \begin{bmatrix} 70 \\ \lambda_1 - 50 \end{bmatrix} \qquad X_2 = \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix}$

$X_1^T X_2 = [70 \quad \lambda_1 - 50] \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix}$

$= 70\,\lambda_2 - 5600 + 70\,\lambda_1 - 3500$

$= 70\,(\lambda_1 + \lambda_2) - 9100 = 70\,(130) - 9100$

$= 9100 - 9100 = \mathbf{0}$

1.42 (b)

$$\begin{bmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{bmatrix}$$

$R_1 \longleftrightarrow R_2 \quad \begin{bmatrix} -1 & -1 & -1 \\ -4 & 1 & -1 \\ 7 & -3 & 1 \end{bmatrix}$

$R_2 - 4R_1, R_3 + 7R_1 \begin{bmatrix} -1 & -1 & -1 \\ 0 & 5 & 3 \\ 0 & -10 & -6 \end{bmatrix}$

$R_3 + 2R_2 \quad \begin{bmatrix} -1 & -1 & -1 \\ 0 & 5 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

No. of non zero rows = 2

rank = **2**

1.43 Sol.

$$A = a\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A| = 4$$

$$|A^{-1}| = \frac{1}{|A|} = \frac{1}{4}$$

1.44 (c)

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Since given matrix is U.T.M.
So, diagonal elements are the eigen values. Hence, $\lambda = 1, 1, 1$. So matrix has only one distinct eigen value i.e.

1.45 (d)

Given system is non-homogeneous system when augmented matrix

$$c = [a/B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ a & -a & 3 & 5 \\ 5 & -3 & a & 6 \end{array}\right]$$

$R_2 \div a$ and $R_3 \to R_3 - 5R_1$

$$c = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & -1 & 3/a & 5/a \\ 0 & -8 & a-5 & 1 \end{array}\right]$$

$R_2 \to R_2 - R_1$,

$$c = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & \frac{3}{a}-1 & \frac{5}{a}-1 \\ 0 & -8 & a-5 & 1 \end{array}\right]$$

$R_3 \to R_3 - 4R_2$,

$$c = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & \frac{3}{a}-1 & \frac{5}{a}-1 \\ 0 & 0 & a-\frac{12}{a}-1 & 5-\frac{20}{a} \end{array}\right]$$

This system will have infinitely many solution only.

If $\quad a-\frac{12}{a}-1 = 0$ and $5-\frac{20}{a} = 0$

$a = -3, 4$ and $a = 4$

for $a = 4$, the system has infinite many solution.

1.46 (d)

$$[A]\{x\} = \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 32 \\ 16 \\ 64 \end{bmatrix} = 16\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = 16\{x\}$$

So, one of the eigen value is 16.

1.47 (b)

Matrix multiplication is associative.

1.48 (c)

$$S = \frac{P+P^T}{2}$$

$$V = \frac{P-P^T}{2}$$

$$P = S + V = \begin{pmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{pmatrix}$$

1.49 Sol.

I_{100}

Eigen values of $I \to \underbrace{1, 1, \ldots\ldots\ldots, 1}_{100 \text{ times}}$

Set of distributed eigen value $E = \{1\}$
Number of elements in $E = 1$

1.50 (d)

Given, $\quad AP = \alpha^2 P$
By comparison with $AX = \lambda X$
$\Rightarrow \quad \lambda = \alpha^2$
Hence, eigen value of A is α^2, so eigen value of A^2 is α^4.

1.51 (c)

For $n = 2$, matrix are

$$Q = \begin{bmatrix} a & c \\ c & b \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, R = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

$$F(x) = \frac{1}{2}[x_1, x_2]\begin{bmatrix} a & c \\ c & b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - [r_1\ r_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \frac{1}{2}\left[ax_1^2 + bx_2^2 + 2cx_1x_2\right] - [r_1x_1 + r_2x_2]$$

i.e. $F(x_1, x_2) = \frac{1}{2}ax_1^2 + \frac{1}{2}bx_2^2 + cx_1x_2 - r_1x_1 - r_2x_2$

Now, for critical point, $\frac{\partial f}{\partial x_1} = 0$ and $\frac{\partial f}{\partial x_2} = 0$

$\Rightarrow \quad a_1x_1 + cx_2 - r_1 = 0$